47) $F''(x) = \frac{2}{x^2}$ $\quad F'(1) = 1$ $\quad F(1) = 1$ $\quad x > 0$

$$\int \frac{d^2x}{dy^2} = \int \frac{2}{x^2}\,dx$$

$\boxed{\begin{matrix} 2x^{-2} \\ -2x^{-1} \end{matrix}}$

$$\frac{dx}{dy} = \frac{-2}{x} + C$$

$$1 = -\frac{2}{1} + C$$

$$3 = C$$

$$\int \frac{dy}{dx} = \int \left(-\frac{2}{x} + 3\right) dx$$

$$y = -2\ln x + 3x + C$$

$$1 = -2\ln(1) + 3(1) + C$$

$$-2 = C$$

$$\boxed{y = -2\ln x + 3x - 2}$$

55) $$\int_1^e \frac{(1+\ln x)^2}{x}\,dx$$

$u = 1 + \ln x$

$du = \frac{1}{x}dx$

Change the limits to u values.

$$= \int_1^2 u^2\,du = \frac{1}{3}u^3\Big|_1^2 = \frac{1}{3}(8-1) = \boxed{\frac{7}{3}}$$

75) $y = \tan x$

$$\int_0^{\frac{\pi}{4}} \tan x\,dx = -\ln|\cos x|\Big|_0^{\frac{\pi}{4}}$$

$$= -\ln\frac{1}{\sqrt{2}} + \ln 1$$

ln(1) = 0

Negative power means reciprocal.

$$= \boxed{\ln\sqrt{2}}$$

alternative answer with the half power (radical) in front.

$$= \boxed{\frac{1}{2}\ln 2}$$

77) $y = x + \frac{4}{x}$

$$\int_1^4 \left(x + \frac{4}{x}\right) dx$$

$$\frac{1}{2}x^2 + 4\ln x\Big|_1^4 = \left[\frac{1}{2}(4)^2 + 4\ln 4\right] - \left[\frac{1}{2}(1)^2 + 4\ln 1\right]$$

$$= 8 + 4\ln 4 - \frac{1}{2}$$

$$\frac{15}{2} + 8\ln 2$$

81) $$\int_1^5 \frac{12}{x}\,dx$$

$n = 4$

$x_0 = 1$

$x_1 = 2$

$x_2 = 3$ $\quad \Delta x = 1$ (h)

$x_3 = 4$

$x_4 = 5$

$\frac{1}{2}(h)(b_1 + b_2)$

$$= \frac{1}{2}(1)(12+6) + \frac{1}{2}(1)(6+4) + \frac{1}{2}(1)(4+3) + \frac{1}{2}(1)(3+1)$$

$$= \frac{1}{2}(39) = \boxed{\frac{39}{2}}$$